



$\text{Pr}(\text{You Are a Bayesian}) > 0.50$

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Lilly



A Problem of Inference



Lilly

A Problem of Inference

10,000 Coins



9,999 Fair Coins (H/T)
1 Biased Coin (H/H)

Problem

1. I draw out one coin.
2. I will flip it repeatedly, and tell you the result.
3. You tell me when you decide whether I have the **Biased Coin** or not.

The Results

Number of Flips	Result	Biased Coin?
-----------------	--------	--------------

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N
5	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N
5	H	Y or N
6	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N
5	H	Y or N
6	H	Y or N
7	H	Y or N

The Results

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7	H	Y or N
8	H	Y or N

The Results

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9	H	Y or N

The Results

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The Results

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8	H	Y or N
9	H	Y or N
10	H	Y or N

Number of Flips	Result	Biased Coin?
11	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N
5	H	Y or N
6	H	Y or N
7	H	Y or N
8	H	Y or N
9	H	Y or N
10	H	Y or N

Number of Flips	Result	Biased Coin?
11	H	Y or N
12	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
3	H	Y or N
4	H	Y or N
5	H	Y or N
6	H	Y or N
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The Results

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Number of Flips	Result	Biased Coin?
11	H	Y or N
12	H	Y or N
13	H	Y or N
14	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
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The Results

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The Results

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Number of Flips	Result	Biased Coin?
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14	H	Y or N
15	H	Y or N
16	H	Y or N
17	H	Y or N
18	H	Y or N

The Results

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7	H	Y or N
8	H	Y or N
9	H	Y or N
10	H	Y or N

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13	H	Y or N
14	H	Y or N
15	H	Y or N
16	H	Y or N
17	H	Y or N
18	H	Y or N
19	H	Y or N

The Results

Number of Flips	Result	Biased Coin?
1	H	Y or N
2	H	Y or N
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14	H	Y or N
15	H	Y or N
16	H	Y or N
17	H	Y or N
18	H	Y or N
19	H	Y or N
20	H	Y or N

Two Perspectives

1. What is the probability of seeing N consecutive heads IF I have a fair coin?
2. What is the probability that I selected the biased coin IF I observe N consecutive heads from a coin randomly drawn from a bag of 9,999 fair coins and 1 biased coin?

Two Perspectives

1. Pr (observed data | coin is fair)

H_0 : Coin is fair [i.e. $\text{pr}(\text{heads}) = 0.50$]

H_a : Coin is not fair [i.e. $\text{pr}(\text{heads}) = 1.00$]

$\text{Pr}[N \text{ consecutive heads} \mid \text{fair coin}] = (0.50)^N$

Two Perspectives

1. Pr (observed data | coin is fair)

H_0 : Coin is fair [i.e. $\text{pr}(\text{heads}) = 0.50$]

H_a : Coin is not fair [i.e. $\text{pr}(\text{heads}) = 1.00$]

$\text{Pr}[N \text{ consecutive heads} \mid \text{fair coin}] = (0.50)^N$

Better known as the *p-value*

Frequentist perspective

Frequentist Results

Number of Flips	Result	p-value
1	H	0.500000000
2	H	0.250000000
3	H	0.125000000
4	H	0.062500000
5	H	0.031250000
6	H	0.015625000
7	H	0.007812500
8	H	0.003906250
9	H	0.001953125
10	H	0.000976563

Number of Flips	Result	p-value
11	H	0.000488281
12	H	0.000244141
13	H	0.000122070
14	H	0.000061035
15	H	0.000030518
16	H	0.000015259
17	H	0.000007629
18	H	0.000003815
19	H	0.000001907
20	H	0.000000954

Frequentist Results

Number of Flips	Result	p-value
1	H	0.500000000
2	H	0.250000000
3	H	0.125000000
4	H	0.062500000
5	H	0.031250000
6	H	0.015625000
7	H	0.007812500
8	H	0.003906250
9	H	0.001953125
10	H	0.000976563

Number of Flips	Result	p-value
11	H	0.000488281
12	H	0.000244141
13	H	0.000122070
14	H	0.000061035
15	H	0.000030518
16	H	0.000015259
17	H	0.000007629
18	H	0.000003815
19	H	0.000001907
20	H	0.000000954

Two Perspectives

2. Pr (coin is biased | observed data)

If we have $Pr(A|B)$,

we want to obtain the conditional probability $Pr(B|A)$

Bayes Theorem (1763)*

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

*As formulated by Laplace (1812)

Bayesian Results

Number of Flips	Result	Pr(Biased Coin)
1	H	0.000200
2	H	
3	H	
4	H	
5	H	
6	H	
7	H	
8	H	
9	H	
10	H	

Number of Flips	Result	Pr(Biased Coin)
11	H	
12	H	
13	H	
14	H	
15	H	
16	H	
17	H	
18	H	
19	H	
20	H	

Bayesian Results

Number of Flips	Result	Pr(Biased Coin)
1	H	0.000200
2	H	0.000400
3	H	
4	H	
5	H	
6	H	
7	H	
8	H	
9	H	
10	H	

Number of Flips	Result	Pr(Biased Coin)
11	H	
12	H	
13	H	
14	H	
15	H	
16	H	
17	H	
18	H	
19	H	
20	H	

Bayesian Results

Number of Flips	Result	Pr(Biased Coin)
1	H	0.000200
2	H	0.000400
3	H	0.000799
4	H	0.001598
5	H	0.003190
6	H	0.006360
7	H	0.012639
8	H	0.024968
9	H	0.048711
10	H	0.092897

Number of Flips	Result	Pr(Biased Coin)
11	H	0.170001
12	H	0.290600
13	H	0.450333
14	H	0.621006
15	H	0.766198
16	H	0.867624
17	H	0.929121
18	H	0.963258
19	H	0.981285
20	H	0.990554

A Problem of Inference

100 Coins



99 Fair Coins (H/T)
1 Biased Coin (H/H)

Problem

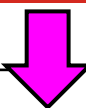
1. I draw out one coin.
2. I will flip it repeatedly, and tell you the result.
3. You tell me when you decide whether I have the **Biased Coin** or not.

The Results

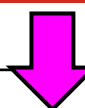
Number of Flips	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)
1	0.000200	0.019802
2	0.000400	0.038835
3	0.000799	0.074766
4	0.001598	0.139130
5	0.003190	0.244275
6	0.006360	0.392638
7	0.012639	0.563877
8	0.024963	0.721127
9	0.048711	0.837971
10	0.092897	0.911843

Number of Flips	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)
11	0.170001	0.953889
12	0.290600	0.976400
13	0.450333	0.988059
14	0.621006	0.993994
15	0.766198	0.996988
16	0.867624	0.998492
17	0.929121	0.999245
18	0.963258	0.999622
19	0.981285	0.999811
20	0.990554	0.999906

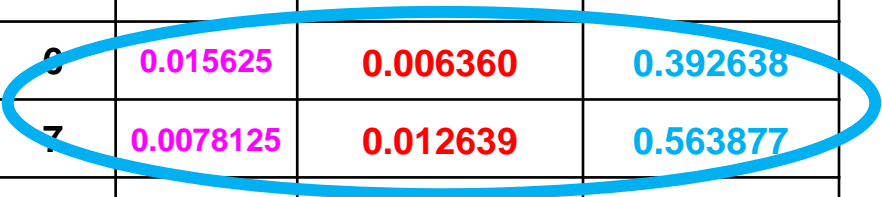
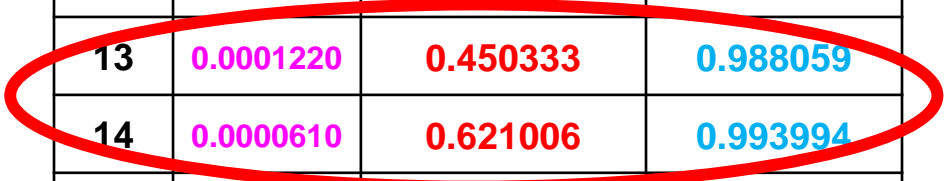
The Results



# of Flips	p-value	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)
1	0.500000	0.000200	0.019802
2	0.250000	0.000400	0.038835
3	0.125000	0.000799	0.074766
4	0.062500	0.001598	0.139130
5	0.031250	0.003190	0.244275
6	0.015625	0.006360	0.392638
7	0.0078125	0.012639	0.563877
8	0.0039063	0.024963	0.721127
9	0.0019531	0.048711	0.837971
10	0.0009766	0.092897	0.911843



# of Flips	p-value	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)
11	0.0004882	0.170001	0.953889
12	0.0002441	0.290600	0.976400
13	0.0001220	0.450333	0.988059
14	0.0000610	0.621006	0.993994
15	0.0000305	0.766198	0.996988
16	0.0000153	0.867624	0.998492
17	0.0000076	0.929121	0.999245
18	0.0000038	0.963258	0.999622
19	0.0000019	0.981285	0.999811
20	0.0000010	0.990554	0.999906



The Results

# of Flips	p-value	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)	# of Flips	p-value	Prior = 1/10,000 Pr(Biased Coin)	Prior = 1/100 Pr(Biased Coin)
1	0.500000	0.000200	0.019802	11	0.0004882	0.170001	0.953889
2	0.250000					0.600	0.976400
3	0.125000					0.333	0.988059
4	0.062500					0.006	0.993994
5	0.031250					0.198	0.996988
6	0.015625					0.624	0.998492
7	0.0078125					0.121	0.999245
8	0.0039063					0.258	0.999622
9	0.0019531	0.048711	0.837971	19	0.0000019	0.981285	0.999811
10	0.0009766	0.092897	0.911843	20	0.0000010	0.990554	0.999906

Note: The p-value never changes regardless of your prior knowledge!!!!

Summary

- ✓ **Bayesian perspective makes **inference** directly about the question of interest**



A Problem of Prediction

Lilly

A Problem of Prediction



**5% of
Population
have ALK
gene**

A Problem of Prediction

Diagnostic Kit



A Problem of Prediction

Patients



Diagnostic Kit

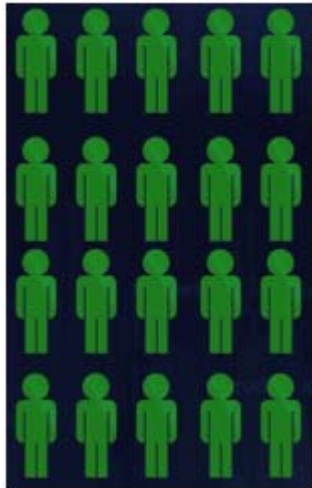


95%
Sensitivity

19 +'s
1 -

A Problem of Prediction

Patients



Diagnostic Kit



**95%
Sensitivity**

**19 +'s
1 -**

**95%
Specificity**

**1 +
19 -'s**

A Problem of Prediction

Individual Patient

ALK(-)?



ALK(+)?

A Problem of Prediction

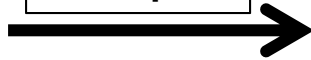
Individual Patient

ALK(-)?



ALK(+)?

Sample



Diagnostic Kit



A Problem of Prediction

Individual Patient

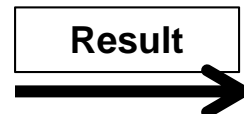
ALK(-)?



ALK(+)?



Diagnostic Kit



A Problem of Prediction

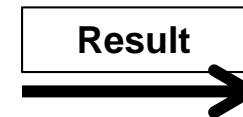
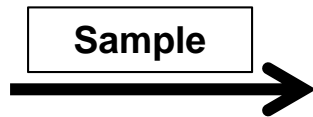
Individual Patient

ALK(-)?



ALK(+)?

Diagnostic Kit



Pr(Patient is ALK+) = ?

Diagnostic Decision-Making

		<i>Patient Characteristic</i>	
		Positive	Negative
Diagnostic Test	Positive		
	Negative		

Diagnostic Decision-Making

		<i>Patient Characteristic</i>	
		Positive	Negative
Diagnostic Test	Positive	True Positive 95% (Sensitivity)	False Positive 5%
	Negative	False Negative 5%	True Negative 95% (Specificity)

Diagnostic Decision-Making

Developing/Designing the "Assay"

		<i>Patient Characteristic</i>	
		Positive	Negative
Diagnostic Test	Positive	True Positive 95% (Sensitivity)	False Positive 5%
	Negative	False Negative 5%	True Negative 95% (Specificity)

Prob (diagnostic test is positive IF the patient has the characteristic)

Diagnostic Decision-Making

Interpreting an Observed Result

		<i>Patient Characteristic</i> (Unknown Truth)		Conditional Probability
		Positive	Negative	
Diagnostic Test	Positive	True Positive 95%	False Positive 5%	Positive Predictive Value
	Negative	False Negative 5%	True Negative 95%	Negative Predictive Value

Prob (patient has the characteristic IF the diagnostic test is positive)

Diagnostic Decision-Making

Underlying Prevalence for ALK gene is **5%**

		<i>Have the ALK Gene</i>		
		Positive (5%)	Negative (95%)	
Diagnostic Test	Positive	True Positive 95%	False Positive 5%	Positive Predictive Value
	Negative	False Negative 5%	True Negative 95%	Negative Predictive Value

Diagnostic Decision-Making

Underlying Prevalence for ALK gene is **5%**

		<i>Have the ALK Gene</i>		
		Positive (5%)	Negative (95%)	
Diagnostic Test	Positive	True Positive 95%	False Positive 5%	Positive Predictive Value
	Negative	False Negative 5%	True Negative 95%	Negative Predictive Value
		100	1900	2000

Diagnostic Decision-Making

Underlying Prevalence for ALK gene is **5%**

		<i>Have the ALK Gene</i>		
		Positive (5%)	Negative (95%)	
Diagnostic Test	Positive	True Positive 95 95%	False Positive 95 5%	Positive Predictive Value
	Negative	False Negative 5 5%	True Negative 1805 95%	Negative Predictive Value
		100	1900	2000

Diagnostic Decision-Making

Underlying Prevalence for ALK gene is **5%**

		<i>Have the ALK Gene</i>		
		Positive (5%)	Negative (95%)	
Diagnostic Test	Positive	True Positive 95 95%	False Positive 95 5%	50% Positive Predictive Value
	Negative	False Negative 5 5%	True Negative 1805 95%	99.7% Negative Predictive Value
		100	1900	2000

Diagnostic Decision-Making

Underlying Prevalence for gene XYZ is **50%**

		<i>Have the XYZ Gene</i>		
		Positive (50%)	Negative (50%)	
Diagnostic Test	Positive	True Positive 95%	False Positive 5%	Positive Predictive Value
	Negative	False Negative 5%	True Negative 95%	Negative Predictive Value

Diagnostic Decision-Making

Underlying Prevalence for gene XYZ is **50%**

		<i>Have the XYZ Gene</i>		
		Positive (50%)	Negative (50%)	
Diagnostic Test	Positive	950 <small>True Positive 95%</small>	50 <small>False Positive 5%</small>	95% <small>Predictive Value</small>
	Negative	50 <small>False Negative 5%</small>	950 <small>True Negative 95%</small>	95% <small>Predictive Value</small>
		1000	1000	2000

Diagnostic Decision-Making

KEY MESSAGES

- ◆ Sensitivity and Specificity are the focus of ***assay design and development***
- ◆ The **Positive (Negative) Predictive Values** are the focus of ***interpreting results*** (assay outputs)
- ◆ **THE PPV (NPV) ARE DEPENDENT ON THE UNDERLYING PREVALENCE OF THE CHARACTERISTIC (e.g. disease/marker status)**

The Clinical Trial Analogy

- ◆ The diagnostic test is the clinical trial
- ◆ The patient characteristic is whether the treatment meets its Critical Success Factors (unknown truth)
- ◆ Sensitivity and (1-Specificity) are analogous to “power” and “significance level” of the hypothesis test
- ◆ The PPV (NPV) is analogous to the “Bayesian posterior probability” that the treatment meets (fails) the CSF
- ◆ **THE PPV (NPV) ARE DEPENDENT ON THE PRIOR PROBABILITY OF THE TREATMENT MEETING THE CSF**

The Clinical Trial Analogy

Entering Ph 2 \Rightarrow Pr(drug meets CSFs) = **20%**

		Meets CSFs		
		Yes (20%)	No (80%)	
CT Result	Positive			Positive Predictive Value
	Negative			Negative Predictive Value

The Clinical Trial Analogy

Entering Ph 2 \Rightarrow Pr(drug meets CSFs) = **20%**

		Meets CSFs		Moderately Rigorous Ph 2 Trial Design
		Yes (20%)	No (80%)	
CT Result	Positive	True Positive 80% ("Power" for a Ph 2 Trial)	False Positive 10% ("Significance Level" for Ph 2)	Positive Predictive Value
	Negative	False Negative 20%	True Negative 90%	Negative Predictive Value
Unknown \rightarrow				

The Clinical Trial Analogy

Entering Ph 2 \Rightarrow Pr(drug meets CSFs) = **20%**

		Unknown \rightarrow <i>Meets CSFs</i>		Moderately Rigorous Ph 2 Trial Design
		Yes (20%)	No (80%)	
CT Result	Positive	True Positive 80% 320	False Positive 10% 160	66.7%
	Negative	False Negative 20% 80	True Negative 90% 1440	94.7%
Observed \uparrow		400	1600	2000

The Clinical Trial Analogy

Entering Ph 2 \Rightarrow Pr(drug meets CSFs) = **20%**

		Unknown \rightarrow <i>Meets CSFs</i>		Moderately Rigorous Ph 2 Trial Design
		Yes (20%)	No (80%)	
CT Result	Positive	True Positive 80% 320 ("Prove it's a Success")	False Positive 10% 160 ("Success")	66.7%
	Negative	False Negative 20% 80 ("Prove it's a Failure")	True Negative 90% 1440 ("Prove it's a Failure")	94.7%
Observed \uparrow		400	1600	2000

"Prior"

"Posterior"

Summary

- ✓ Bayesian posterior probabilities should be used to interpret trial outcomes (i.e. used **predict** the likelihood of events)



A Problem of Evidence

Bayes Factor*

Let p_0 = prior probability that H_0 is false
(i.e. the drug works)

Bayes Factor*

Let p_0 = prior probability that H_0 is false
(i.e. the drug works)

Let p = observed p-value for test statistics for H_0

Bayes Factor*

Let p_0 = prior probability that H_0 is false
(i.e. the drug works)

Let p = observed p-value for test statistics for H_0

Bayes factor* **$[-e \times p \times \ln(p)]$** used to compute upper bound on posterior probability that H_0 is false

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The American Statistician, February 2001, Vol. 55, No. 1, pp 62-71.

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Posterior probability** for H_0 being false is

$$p_1 \leq \{1 + [(1-p_0)/p_0] \times [-e \times p \times \ln(p)]\}^{-1}$$

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**If $p < 1/e = .368$

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```
graph LR; PD[Posterior] --> p1; P[Prior] --> Pterm["[(1-p0)/p0]"]; ND[New Data] --> Bterm["[-e × p × ln(p)]"]; Pterm --- Plus["+"]; Bterm --- Plus; Plus --- Brackets["{}"]; Brackets --- p1;
```

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**If $p < 1/e = .368$

Bayes Factor

- ◆ If your prior is 30% probability of success (i.e. H_0 being false) entering Phase 2 ...

Observed Phase 2 p-Value	Upper Bound on Posterior Probability for H_0 Being False*
0.20	.329
0.10	.406
0.05	.513
0.01	.774

*Using Bayes factor for converting p-values into posterior probabilities

Bayes Factor

- ◆ If your prior is 30% probability of success (H_0 being false) entering Phase 2 ...
- ◆ And you want to exit Phase 2 with an 70% probability of success ...
- ◆ Then you need* ...
 - 1 study with a p-value of 0.016
 - 2 studies each with p-values of 0.05**

*Using Bayes factor for converting p-values into posterior probabilities

**Successive application of Bayes factor

FDA Approval

- ◆ Suppose FDA wants to be 95% sure that H_0 is false
- ◆ Substantial evidence
 - **Consider one-tailed p-values < 0.025 for two Ph 3 CT**

Prior Probability Against H_0 Entering Phase 3	Posterior Probability for H_0 Being False with Two p-values of 0.025 (\leq)
0.50	.941
0.60	.960
0.70	.974

FDA Approval

- ◆ Suppose FDA wants to be 95% sure that H_0 is false
- ◆ Substantial evidence
 - **What p-value is needed for a single Ph 3 CT?**

Prior Probability Against H_0 Entering Phase 3	Posterior Probability for H_0 Being False with Two p-values of 0.025 (\leq)
0.50	.941
0.60	.960
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FDA Approval

- ◆ Suppose FDA wants to be 95% sure that H_0 is false
- ◆ Substantial evidence
 - **What p-value is needed for a single Ph 3 CT?**

Prior Probability Against H_0 Entering Phase 3	Posterior Probability for H_0 Being False with Two p-values of 0.025 (\leq)	Single p-value for 95% Posterior Probability of H_0 Being False (\leq)
0.50	.941	0.003
0.60	.960	0.006
0.70	.974	0.010

FDA Approval

- ◆ So, what's the big deal?

FDA Approval

- ◆ So, what's the big deal?
- ◆ Suppose two Ph 3 CT's result in (two-tailed)
 - $p = 0.02$
 - $p = 0.08$
- ◆ Not Approved!
- ◆ Suppose you have a 0.60 prior that H_0 is false.
 - $p = 0.02$
 - $p = 0.08$
- ◆ Bayes factor \rightarrow $\text{pr}(H_0 \text{ is false}) \leq .972$

A Problem of Evidence

“Is a p-value < 0.05 strong **evidence** against the null hypothesis?”

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- ✓ It depends!
 - ✓ ... on your prior belief
 - ✓ ... on how small the p-value is

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- ✓ ... on how small the p-value is

- ✓ Very Bayesian notions of substantial evidence !!!

Summary

- ✓ Bayesian probabilities quantify **evidence** related to a hypothesis of interest

Conclusion

- A p-value is not interpretable without understanding prior belief about the hypothesis of interest
 - + It is more amenable to a black and white rule for decision-making

Conclusion

- A p-value is not interpretable without understanding prior belief about the hypothesis of interest
 - + It is more amenable to a black and white rule for decision-making
- + A Bayesian posterior probability provides the likelihood of a hypothesis being true/false
 - It requires more sophisticated, flexible use of data for decision-making



Pr(I Thank You) = 1.00

Lilly